AE 424 Fall 2021 Group 5 Final Project: Thin Airfoil Theory

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This report aims to compare analysis methods for airfoils. By selecting "Airfoil A" from the problem statement we analyzed the airfoil using thin airfoil theory and a program called JavaFoil. The results of analysis are then compared to experimental data and the error between methods discussed. Thin airfoil theory and JavaFoil are found to have similar results, while experimental data strays from the analysis trends.

Nomenclature

α	=	angle of attack (degrees)
c_l	=	lift coefficient
c_m	=	moment coefficient
c_p	=	pressure coefficient
Re	=	Reynold's Number
A_n	=	thin airfoil theory Taylor series coefficients
$\frac{\partial}{\partial\alpha}$	=	partial derivative with respect to α

I. Introduction

THIN airfoil theory (TAT) as described in AE 424 is the mathematical analysis of airfoil properties wherein the airfoil is represented by a vortex sheet placed on the mean camber line of the airfoil. Given four airfoils, this group decided to analyze airfoil A with given properties to be described. This report attempts to compare analysis methods for airfoil. These methods: experimental given data, JavaFoil, and thin airfoil theory.

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II. Procedure

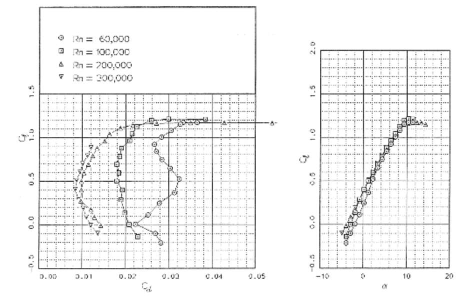
A. Experimental Data

Experimental Data is given by the following figures.

Fig.1: Experimental Surface Data for Airfoil A

1	1.00000	0.00000	19	0.25000	0.08996	37	0.00428	-.00898	55	0.37059	-.02242
2	0.99572	0.00115	20	0.22221	0.08774	38	0.00961	-.01296	56	0.43474	-.02018
3	0.98296	0.00448	21	0.19562	0.08483	39	0.01704	-.01651	57	0.50000	-.01792
4	0.96194	0.00972	22	0.17033	0.08113	40	0.02653	-.01959	58	0.56526	-.01566
5	0.93301	0.01656	23	0.14645	0.07660	41	0.03806	-.02214	59	0.62941	-.01345
6	0.89668	0.02475	24	0.12408	0.07134	42	0.05156	-.02414	60	0.69134	-.01131
7	0.85355	0.03400	25	0.10332	0.06552	43	0.06699	-.02567	61	0.75000	-.00928
8	0.80438	0.04394	26	0.08427	0.05939	44	0.08427	-.02680	62	0.80438	-.00741
9	0.75000	0.05412	27	0.06699	0.05313	45	0.10332	-.02763	63	0.85355	-.00575
10	0.69134	0.06405	28	0.05156	0.04677	46	0.12408	-.02816	64	0.89668	-.00429
11	0.62941	0.07319	29	0.03806	0.04027	47	0.14645	-.02839	65	0.93301	-.00302
12	0.56526	0.08105	30	0.02653	0.03352	48	0.17033	-.02832	66	0.96194	-.00190
13	0.50000	0.08719	31	0.01704	0.02652	49	0.19562	-.02795	67	0.98296	-.00094
14	0.43474	0.09128	32	0.00961	0.01943	50	0.22221	-.02734	68	0.99572	-.00025
15	0.37059	0.09312	33	0.00428	0.01254	51	0.25000	-.02653	69	1.00000	0.00000
16	0.33928	0.09318	34	0.00107	0.00616	52	0.27886	-.02559			
17	0.30866	0.09266	35	0.00000	0.00047	53	0.30866	-.02458			
18	0.27886	0.09158	36	0.00107	-.00453	54	0.33928	-.02351			

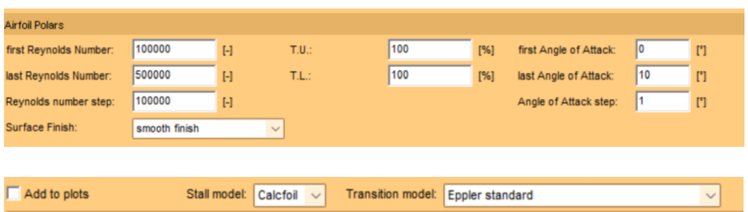
Fig 2: Experimental Lift and Drag Data for Airfoil A



B. JavaFoil

JavaFoil settings are as follows

Fig 3: JavaFoil Configuration Settings



C. MATLAB Code

```
function Airfoil(fileName, pts, figNum)

if nargin < 2
    POINTS_DEFAULT = 61;
    pts = POINTS_DEFAULT;
end
if nargin < 3
    FIGURE_DEFAULT = 1;
    figNum = FIGURE_DEFAULT;
end

fileID = fopen(fileName, 'r');
data = fscanf(fileID, '%f',[2, pts]);

x = data(1,:);
y = data(2,:);

camber = zeros(1,(pts-1)/2);

for i = 1:((pts-1)/2)
    camber(1,i) = ( y(1,i) + y(1, (pts+1)-i) ) / 2;
end

figure(figNum);
plot(x,y,x(1,1:(pts-1)/2),camber);
axis([0 1 -0.25 0.25]);
end
```

III.Results and Data

This section contains the data from the analysis methods following the procedure as outlined above.

D. Experimental Givens

Refer to section II.A for the experimental data on Airfoil A.

E. Thin Airfoil Theory

An important note about Thin Airfoil Theory calculations to consider is that the coefficient of the moment of about the quarter chord ($c_{m,c/4}$) is not a function of α. Based on written calculations $c_{m,c/4}$ = -0.066178 or -0.07332 always according to written calculations I and II respectively.

Fig. 4: Thin Airfoil Written Calculations 1

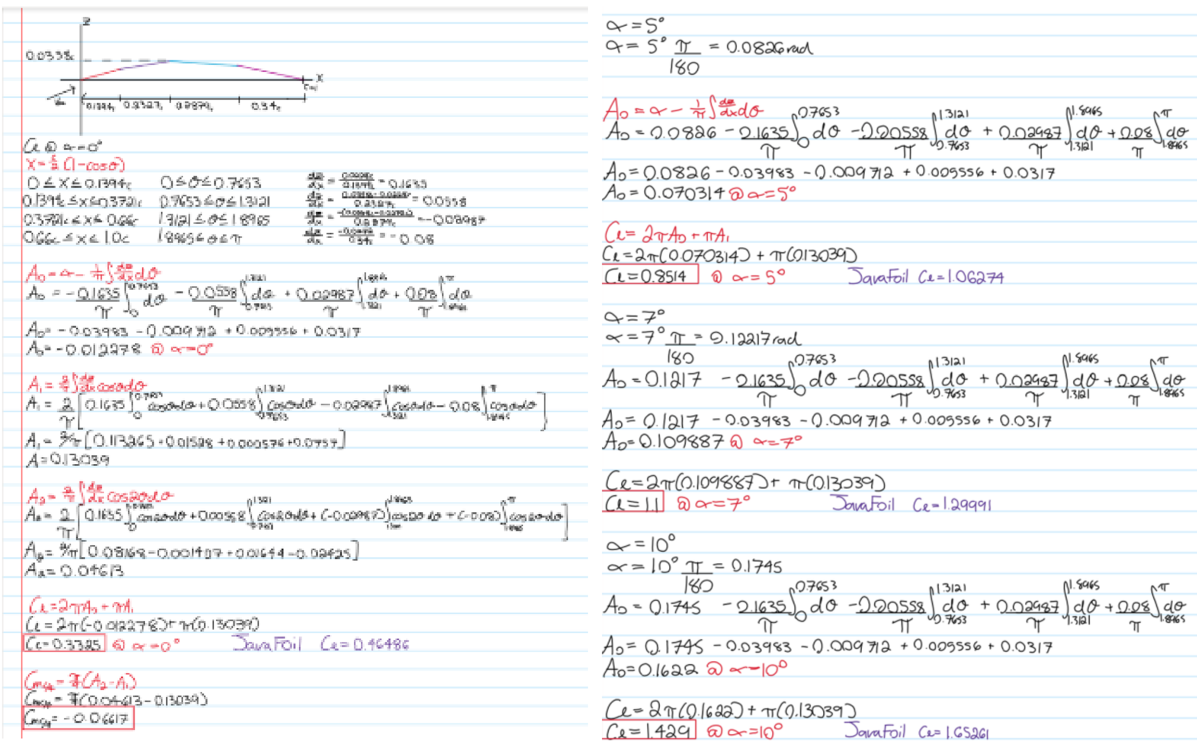
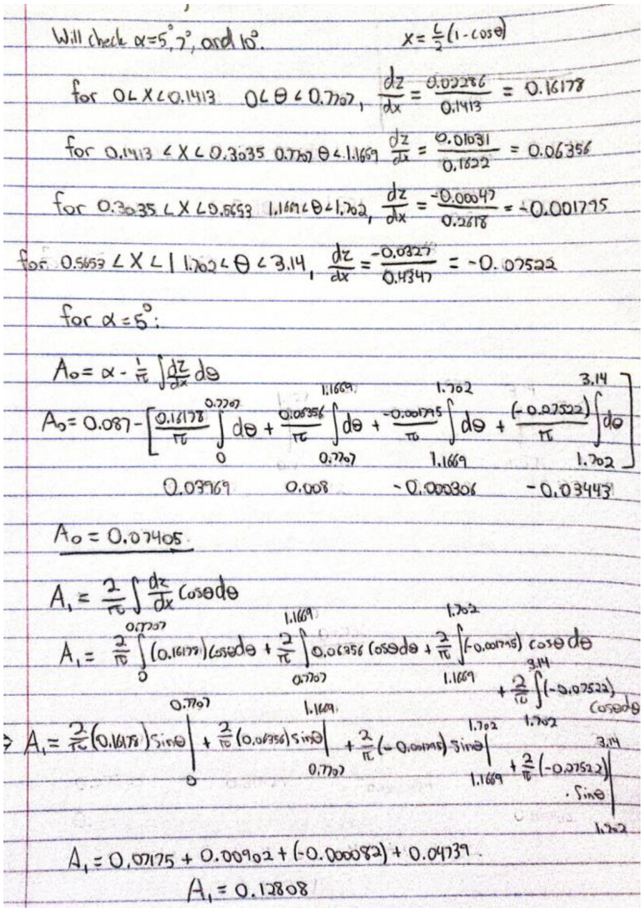


Fig. 5: Thin Airfoil Written Calculations 2



Will check $\alpha = 5°, 7°$, and $10°$. $x = \frac{c}{2}(1-\cos\theta)$

for $0 < x < 0.1413$, $0 < \theta < 0.7707$, $\frac{dz}{dx} = \frac{0.02286}{0.1413} = 0.16178$

for $0.1413 < x < 0.3035$, $0.7707 < \theta < 1.1669$, $\frac{dz}{dx} = \frac{0.01031}{0.1622} = 0.06356$

for $0.3035 < x < 0.5653$, $1.1669 < \theta < 1.702$, $\frac{dz}{dx} = \frac{-0.00047}{0.2618} = -0.001795$

for $0.5653 < x < 1$, $1.702 < \theta < 3.14$, $\frac{dz}{dx} = \frac{-0.0327}{0.4347} = -0.07522$

for $\alpha = 5°$:

$A_0 = \alpha - \frac{1}{\pi}\int \frac{dz}{dx} d\theta$

$A_0 = 0.087 - \left[\frac{0.16178}{\pi}\int_0^{0.7707} d\theta + \frac{0.06356}{\pi}\int_{0.7707}^{1.1669} d\theta + \frac{-0.001795}{\pi}\int_{1.1669}^{1.702} d\theta + \frac{(-0.07522)}{\pi}\int_{1.702}^{3.14} d\theta\right]$

0.03969 0.008 −0.000306 −0.03443

$A_0 = 0.07405$

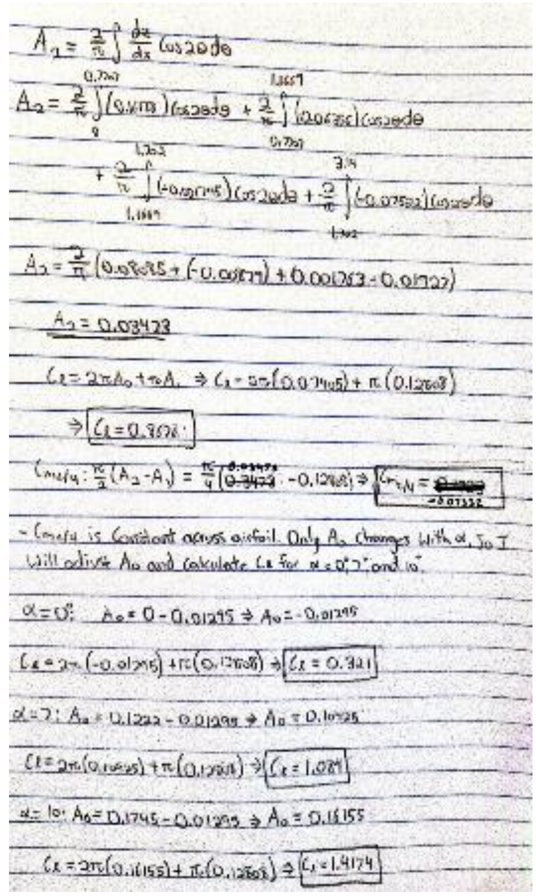
$A_1 = \frac{2}{\pi}\int \frac{dz}{dx}\cos\theta d\theta$

$A_1 = \frac{2}{\pi}\int_0^{0.7707}(0.16178)\cos\theta d\theta + \frac{2}{\pi}\int_{0.7707}^{1.1669} 0.06356\cos\theta d\theta + \frac{2}{\pi}\int_{1.1669}^{1.702}(-0.001795)\cos\theta d\theta + \frac{2}{\pi}\int_{1.702}^{3.14}(-0.07522)\cos\theta d\theta$

$\Rightarrow A_1 = \frac{2}{\pi}(0.16178)\sin\theta\Big|_0^{0.7707} + \frac{2}{\pi}(0.06356)\sin\theta\Big|_{0.7707}^{1.1669} + \frac{2}{\pi}(-0.001795)\sin\theta\Big|_{1.1669}^{1.702} + \frac{2}{\pi}(-0.07522)\cdot\sin\theta\Big|_{1.702}^{3.14}$

$A_1 = 0.07175 + 0.00902 + (-0.000082) + 0.04739$

$A_1 = 0.12808$



$A_2 = \frac{2}{\pi}\int \frac{dz}{dx}\cos 2\theta d\theta$

$A_2 = \frac{2}{\pi}\int_0^{0.7707}(0.16178)\cos 2\theta d\theta + \frac{2}{\pi}\int_{0.7707}^{1.1669}(0.06356)\cos 2\theta d\theta + \frac{2}{\pi}\int_{1.1669}^{1.702}(-0.001795)\cos 2\theta d\theta + \frac{2}{\pi}\int_{1.702}^{3.14}(-0.07522)\cos 2\theta d\theta$

$A_2 = \frac{2}{\pi}(0.08085 + (-0.05879) + 0.001263 - 0.01727)$

$A_2 = 0.03423$

$C_l = 2\pi A_0 + \pi A_1 \Rightarrow C_l = 2\pi(0.07405) + \pi(0.12808)$

$\Rightarrow C_l = 0.8676$

$C_{m_{c/4}}: \frac{\pi}{4}(A_2 - A_1) = \frac{\pi}{4}(0.03423 - 0.12808) \Rightarrow C_{m_{c/4}} = -0.07332$

- $C_{m_{c/4}}$ is constant across airfoil. Only A_0 changes with α, so I will adjust A_0 and calculate C_l for $\alpha = 0°, 7°$, and $10°$.

$\alpha = 0°$: $A_0 = 0 - 0.01295 \Rightarrow A_0 = -0.01295$

$C_l = 2\pi(-0.01295) + \pi(0.12808) \Rightarrow C_l = 0.321$

$\alpha = 7°$: $A_0 = 0.1222 - 0.01295 \Rightarrow A_0 = 0.10925$

$C_l = 2\pi(0.10925) + \pi(0.12808) \Rightarrow C_l = 1.084$

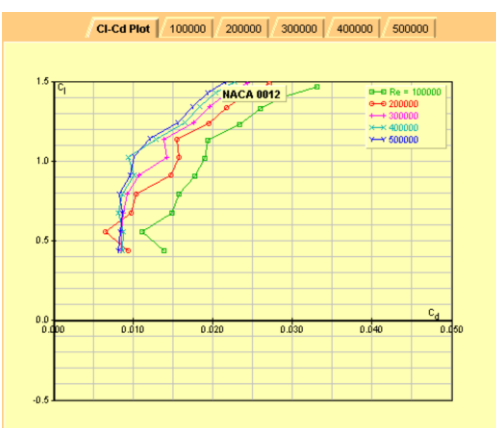
$\alpha = 10°$: $A_0 = 0.1745 - 0.01295 \Rightarrow A_0 = 0.16155$

$C_l = 2\pi(0.16155) + \pi(0.12808) \Rightarrow C_l = 1.4174$

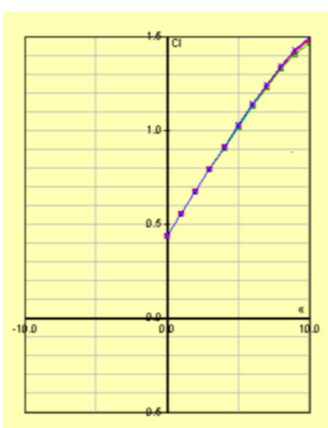
F. JavaFoil

Figure Set 1: Graphical Data for JavaFoil

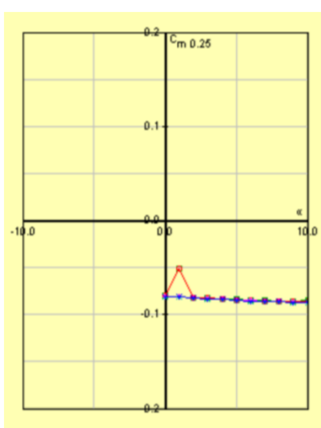
A: cl-cd Curves



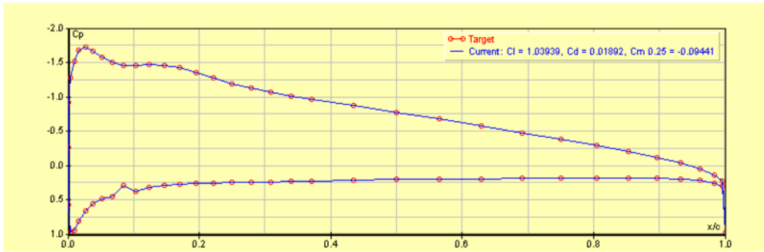
B: c_l – α Curves



C: c_m – α Curves



D: c_p curve at $\alpha = 5°$



Using trapezoidal approximation of the integral for c_l from $c_{p,5°}$:

$$c_l = 1.0355$$

G. MATLAB

This section contains the results of the MATLAB code found in the **II** section. MATLAB polyfit results are the foundation of Fig. 3 and Fig. 4.

Fig. 6: Plotted Airfoil and Estimation

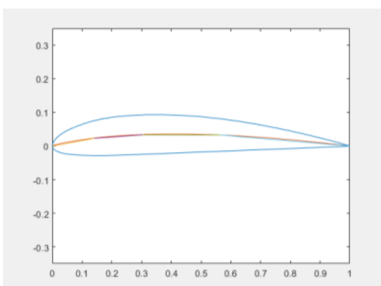
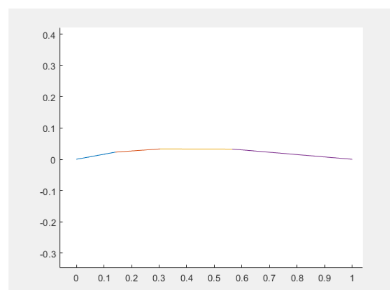


Fig. 7: Isolated MCL Estimation



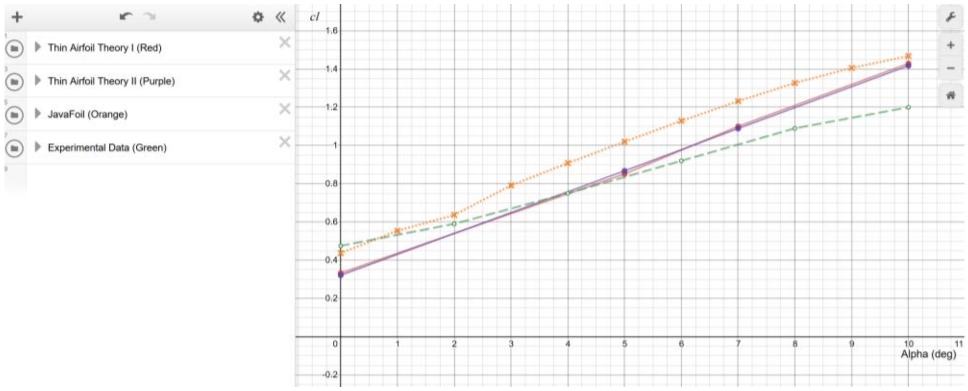
y-axis: Distance from Chord Line in units of Chord

x-axis: Chord Line in units of of Chord

IV. Discussion of Differences

H. Plotted Differences

Fig. 8: Comparison of Experimental Data, JavaFoil, and Thin Airfoil Theory



I. Discussion

As α varies from 0° to 10°, the value of c_l is dependent on the method of analysis. Experimental data is likely from extensive wind tunnel testing and an accurate representation of actual c_l values. Error may be found in the reading of the plots, but the values are given to the group. JavaFoil data for Re = 100000 are plotted in the figure. JavaFoil has error associated with computational methods inside of the program and in choice of Re, but compared to higher Re, the values remain exact within .01 of values in the figure. Thin Airfoil Theory results follow the trend of JavaFoil more closely than that of the experimental data, but also represents a smaller data set.

The differences between analysis method can be found entirely within the error associated with the approximations and assumptions made. Thin Airfoil Theory does not consider Re, whereas JavaFoil does. Experimental data contains the observation of frictional forces, whereas TAT assumes inviscid flow: helping to explain the trend of decreasing $\frac{\partial c_l}{\partial \alpha}$ in the experimental data. It should also be noted that the experimental data nears stall near α = 10°, further describing the decline.

V.References

"Desmos Graphing Calculator," *Desmos, Inc*, 2021.

Hepperle, M., "JavaFoil," *MH Aero Tools*, 2018.

"MATLAB R2021b – for student use," *MathWorks*, 2021.